Maths Learning Service: Revision **Mathematics IA**

Differentiation **Mathematics IMA**

The derivative

Consider a function $y = f(x)$. For some point x, we can find

- the slope of the tangent to the curve described by $f(x)$, or
- the instantaneous rate at which y is changing

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$f(x)$	$f\ (x)$
k (a constant)	0

Exercises

(1) For $f(x) = x^2$ show that $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = 2x$.

(2) Differentiate $y = x^2(2x - 1)$ with respect to x by

(a) expanding the RHS first, and
(b) the product rule.

(3) Differentiate the following functions with respect to x:

(a) $y = x^4 + x^{-4} + 4$ (b) $f(x) = \dfrac{1}{x}$

(c) $y = 6x^{-\frac{2}{3}}$ (d) $f(x) = 4x^3e^x$

(e) $\sqrt{x}(3x - 1)$ [recall that $\sqrt{x} = x^{\frac{1}{2}}$]

(f) $y = e^x + (x^4 + 1)\ln x + 5$ (g) $f(x) = x^5(x^2 + 6)(x + e^x)$

(h) $y = ax^2 + bx + c$ where a, b and c are constants

(i) $f(x) = a^3 + a^2b + ab^2 + b^3$ where a and b are constants

(j) $y = \dfrac{1 + 3x}{2 - x}$ (k) $f(x) = \dfrac{x^2 - 3x + 1}{x + 2}$

(l) $y = \dfrac{(x + 1)e^x}{x}$ (m) $f(x) = \dfrac{\sqrt{x}}{5x + 2}$

(n) $y = \dfrac{1}{6x^2 + 7}$

The Chain Rule

This is the most useful rule of the lot and is based on the following idea:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx},$$

where u is a function of x that suits you.

Example: $y = e^{3x}$ can't be differentiated by the current rules but it could be done if $u(x) = 3x$ and we apply the chain rule.

$$y = e^u \quad \text{so} \quad \frac{dy}{du} = e^u \quad \text{and} \quad \frac{du}{dx} = 3.$$

Hence

[redacted] $3 = 3e^{3x}$.

x4 dy xe 2 x 3

Example: Consider $y = (2x +$ [redacted]

xe 2 x y