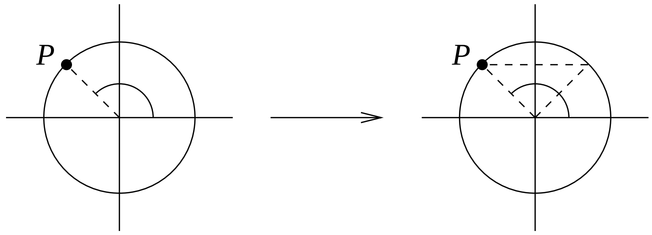
Maths Learning Service: Revision ***Mathematics IA***

More Trigonometry

$$\cos\frac{\pi}{2} = x - \text{co-ordinate of } P = 0.$$

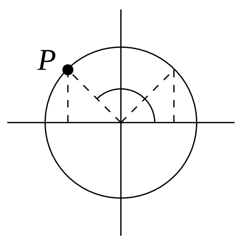
Check these results on a calculator.

Example: What are $\sin\frac{3\pi}{4}$, $\cos\frac{3\pi}{4}$ and $\tan\frac{3\pi}{4}$?



By symmetry, the $y-$co-ordinate of P is the same as for the first quadrant angle $\frac{\pi}{4}$. Hence

$$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$



The $x-$co-ordinate of P is the negative of that for the first quadrant angle $\frac{\pi}{4}$. Hence

$$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}.$$

Finally,

$$\tan\frac{3\pi}{4} = \frac{1/\sqrt{2}}{-1/\sqrt{2}} = -1.$$

Exercises

1. Find the values of

   (a) $\sin 0$, $\sin\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right)$, $\sin\pi$, $\sin\frac{3\pi}{2}$, $\sin 2\pi$.

   (b) $\cos 0$, $\cos\frac{\pi}{2}$, $\cos\left(-\frac{\pi}{2}\right)$, $\cos\pi$, $\cos\frac{3\pi}{2}$, $\cos 2\pi$.

   (c) $\tan 0$, $\tan\pi$, $\tan\frac{3\pi}{4}$, $\tan\frac{7\pi}{4}$, $\tan\left(-\frac{\pi}{4}\right)$.

   (d) $\sin\frac{7\pi}{6}$, $\cos\frac{7\pi}{6}$, $\tan\frac{7\pi}{6}$.

(e) $\sin \frac{11\pi}{6}$, $\cos \frac{11\pi}{6}$, $\tan \frac{11\pi}{6}$.

2. Use the unit circle to show that

(a) $\sin(\pi - \theta) = \sin\theta$ (b) $\cos(\pi - \theta) = -\cos\theta$ (c) $\tan(\pi + \theta) = \tan\theta$

(d) $\cos(2\pi - \theta) = \cos\theta$

Circular Functions

Many situations are affected by circular motion (eg. day length in temperate areas such as Adelaide) and can be modelled using functions of the form

$$y = \sin x, \quad y = \cos x \quad \text{or} \quad y = \tan x.$$

Plotting points from the unit circle produces these distinctive graphs (where x is measured in radians):

$y = \sin x$

Note: The shape of the cosine graph is the same as for sine, but shifted backwards $\frac{\pi}{2}$ units along the x−axis.

y

x

$(0,0)$ 2π

$(\frac{\pi}{2}, D)$

Trigonometric Equations

Example: Find all solutions to $2\sin x + \sqrt{3} = 0$.

Re-arranging this equation gives

$$\sin x = -\frac{\sqrt{3}}{2}.$$

We know that sin $^{\pi}$