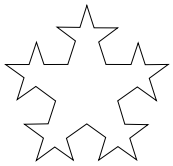
Maths Learning Service: Revision

Index Laws

Mathematics IA

Mathematics IMA

Intro. to Fin. Maths I



Index laws are the rules for simplifying expressions involving *powers* of the same *base* number.

$a^m \times a^n = a^{m+n}$	First Index Law
$(a^m)^n = a^{mn}$	Second Index Law
$\frac{a^m}{a^n} = a^{m-n}$	Third Index Law
$a^{-m} = \frac{1}{a^m}$	
$a^0 = 1$	
$a^{\frac{1}{n}} = \sqrt[n]{a}$	

Examples: Simplify the following expressions, leaving only positive indices in the answer.

(a) $\frac{3^6 2^4}{3^4}$

$= \frac{3^6}{3^4} \times 2^4$

$= 3^2 2^4$

(b) $3^2 \times 3^{-5}$

$= 3^{-3}$

$= \frac{1}{3^3}$

$= \frac{1}{27}$

(c) $\frac{9(x^2)^3}{3xy^2}$

$= \frac{9}{3} \times \frac{x^6}{x} \times \frac{1}{y^2}$

$= 3 \times x^5 \times \frac{1}{y^2}$

$= \frac{3x^5}{y^2}$

(d) $a^{-1}\sqrt{a}$

$= a^{-1}a^{\frac{1}{2}}$

$= a^{-\frac{1}{2}}$

$= \frac{1}{a^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{a}}$

Notes: (1) More involved fractional powers can be dealt with by noting that $\boxed{a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m}$ by the Second Index Law. For example,

$$(27)^{\frac{2}{3}} = (27^{\frac{1}{3}}$$

(3) In general $\boxed{(ab)^n = a^n b^n}$. For example,

$$(3x^2y)^3 = 3^3(x^2)^3y^3 = 27x^6y^3.$$

Exercises

1. Simplify the following expressions, leaving only positive indices in the answer.

(a) $4^2 \times 4^{-3}$ (b) $\dfrac{3^2\,(2^2)^{-2}}{2^3}$ (c) x^5x^8

(d) $(y^4)^6$ (e) $(-3)^3$ (f) $(4ab^2c)^3$

(g) $x^2z^{-3} \times (xz^2)^2$ (h) $2^n \times (2^{-n})^3 \times 2^{2n}$ (i) $3^m \times 27^m \times 9^{-m}$

(j) $(a^{\frac{1}{2}} \times a)^5$ (k) $\dfrac{(-2ab)^2}{2b}$ (l) $\dfrac{(-a^4b)^3(ab)^5}{-a^8b^8}$

(m) $\dfrac{x^{-1}y^4}{x^{-5}y^{-3}}$ (n) $\left(\dfrac{10a^3b^{-2}}{5a^{-1}b^2}\right)^{-1}$ (o) $x\sqrt[3]{}$

$$\begin{aligned}
\frac{\sqrt{3}}{\sqrt{2}} + \frac{2}{\sqrt{6}} &= \frac{\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{2}} + \frac{2}{\sqrt{6}} \\
&= \frac{3}{\sqrt{6}} + \frac{2}{\sqrt{6}} \\
&= \frac{5}{\sqrt{6}} \\
&= \frac{5}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\
&= \frac{5\sqrt{6}}{6}
\end{aligned}$$

Answers to Exercises

1. (a) $\frac{1}{4}$ (b) $\frac{9}{2^7} = \frac{9}{127}$ (c) x^{13} (d) y^{24} (e) -27

   (f) $64a^3b^6c^3$ (g) x^4z (h) 1 (i) 3^{2m} (j) $a^{15/2}$

   (k) $2a^2b$ (l) a^9 (m) x^4y^7 (n) $\frac{1}{2}a^{-4}b^4$ (o) $x^{4/3}$

   (p) a^5 (q) $2x^{-1/2}$ (r) a^{-2} (s) 8 (t) $\frac{8}{125}$

   (u) 2

2. (a) $5\sqrt{2}$ (b) $6\sqrt{2}$ (c) $5\sqrt{3}$ (d) $\frac{2\sqrt{5}-\sqrt{10}}{10}$

   (e) $\frac{5\sqrt{6}}{6}$ (f) $\frac{5\sqrt{3}-6\sqrt{5}}{15}$